Applications of

The Category Theory of

Topological Thermodynamics

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Applications of

The Category Theory of Topological Thermodynamics

This article was motivated in part by the challenge of the Clay Institute regarding the properties of the Navier-Stokes equations and their relationship to hydrodynamic turbulence. To replicate a statement made by the Clay Institute:

"The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations."
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will prove that the Navier-Stokes equations have Turbulent solutions
The method will be to use the abstract **Category Theory of Topological Thermodynamics** for a non-equilibrium particle system, and show that there exists a homotopic evolution of the system topology for at least one specific process that is topologically equivalent to a thermodynamic **irreversible** process, $T_4$. 
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Then it will be shown that there are specific choices of functions that permit the cohomological statement of the First Law of Thermodynamics to be put into 1-1 correspondence with the functions that define the Navier-Stokes equations.
The method will be to use the abstract **Category Theory of Topological Thermodynamics** for a non-equilibrium particle system, and show that there exists a homotopic evolution of the system topology for at least one specific process that is topologically equivalent to a thermodynamic **irreversible** process, $T_4$.

Then it will be shown that there are specific choices of functions that permit the cohomological statement of the First Law of Thermodynamics to be put into 1-1 correspondence with the functions that define the Navier-Stokes equations.

Finally, the abstract irreversible process, $T_4$, will be evaluated in terms of the Navier-Stokes functions, thereby proving that there is at least one solution which is irreversible, and describes a **turbulent process**.
What is Turbulence?

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Turbulence (more or less) is (intuitively) a time dependent motion of a continuum fluid which is the antithesis of a streamline flow.

The fluid motion may or may not be chaotic or self-similar.

However there is a BOTTOM LINE:
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However there is a BOTTOM LINE:

Almost everyone will agree that Turbulence involves an Irreversible Thermodynamic Process.
How do you detect Turbulence?

Turbulence is a process in a topological space of Pfaff Topological dimension 4 that involves exchange of radiation (waves) and matter (particles).
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However, turbulence is visually detected by particle-like topologically coherent defects of Pfaff Topological dimension 3 or less embedded in the topological 4D environment.
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However, turbulence is visually detected by particle-like topologically coherent defects of Pfaff Topological dimension 3 or less embedded in the topological 4D environment.

If I have time I will display various topological defects Embedded in the topological 4D environment.
The **Category theory of Topological Thermodynamics** and any exterior differential 1-form, $A$, of rank 4, can be used to

1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.
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2. generate a Jacobian correlation matrix, $\left[\frac{\partial A_k}{\partial x^k}\right]$ that has a singular set of rank 3, and which is a morphism of a universal van der Waals gas.
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1. generate a disconnected Cartan Topology that defines a non-equilibrium thermodynamic system of particles.

2. generate a Jacobian correlation matrix, \( [\partial A_k / \partial x^k] \) that has a singular set of rank 3, and which is a morphism of a universal van der Waals gas.

3. generate a unique process current of Topological Torsion, \( T = A \wedge dA \), which describes an irreversible thermodynamic process in a non-equilibrium system.
The **Category theory of Topological Thermodynamics**, with homotopic morphisms mapping topological structures $A \Rightarrow Q$ produces a universal topological:

**FIRST LAW OF THERMODYNAMICS**

$$L(J)A = i(J)dA + d\{i(J)A\} \Rightarrow Q$$
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\[
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The Lie differential of the Action 1-form, $A$, (relative to the process, $J$) generates the inexact 1-form of Work, $W$, plus the differential of the Internal energy, $dU$, which is equal to the inexact 1-form of Heat $= Q$. 

\[
L(J)A = W + d\{U\} \Rightarrow Q
\]
The Homotopy operator relative to a process $J$ acting on exterior differential forms is given by Cartan's Magic formula

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Consider a process $J$ such that

$$J = A^dA = i(T_4)dx^dy^dz^dt$$

“The Topological Torsion 3-form”
**Topological Torsion Properties**

\[ T_4 \text{ on } \Omega_4 : \text{Properties of Topological Torsion} \]

\[ i(T_4)\Omega_4 = i(T_4)dx^\wedge dy^\wedge dz^\wedge dt = A^\wedge dA, \]

\[ i(T_4)i(T_4)\Omega_4 = 0, \text{ which implies that} \]

Work 1-form \[ W = i(T_4)dA = \sigma A, \]

\[ dW = d\sigma^\wedge A + \sigma dA = dQ, \]

Internal Energy \[ U = i(T_4)A = 0, \quad T_4 \text{ is associative,} \]

\[ i(T_4)dU = 0 \]

\[ i(T_4)Q = 0 \quad \text{ T}_4 \text{ is adiabatic} \]

\[ L(T_4)A = \sigma A, \quad T_4 \text{ is homogeneous and self-similar} \]

\[ L(T_4)dA = d\sigma^\wedge A + \sigma dA = dQ, \]

\[ Q^\wedge dQ = L(T_4)A^\wedge L(T_4)dA = \sigma^2 A^\wedge dA \neq 0, \quad T_4 \text{ is irreversible,} \]

\[ dA^\wedge dA = d(A^\wedge dA) = d\{(i(T_4)\Omega_4) = (div_4T_4)\Omega_4, \]

\[ L(T_4)\Omega_4 = d\{(i(T_4)\Omega_4) = (2\sigma)\Omega_4, \quad T_4 \text{ causes } \Omega_4 \text{ expansion} \]}
To be thermodynamically irreversible, a process $J$ must

1. Create a heat 1-form, $Q$, that (because of shear viscosity) is chaotic, not integrable and of $\text{PTD}(Q)>2$:

\[ Q^dQ \neq 0 \]

2. and the 3-form $A^dA$ should not be closed. (due to bulk viscosity of expansion-contraction) $\text{PTD}(A) = 4$

\[ d(A^dA) \neq 0 \]
Topological Torsion $A^dA$ is a key design tool for controlling and understanding Dissipative Structures and TURBULENCE
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Almost NO Engineers and Very Few Physicists Understand TOPOLOGICAL TORSION (pity)
The next step is to use the Topological First Law as an equation of homotopic evolution.

\[ \mathcal{L}_{(J)A} = \mathbf{W} + d\{U\} = Q \]
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\[ L_{(J)}A = W + d\{U\} = Q \]

and deduce a functional choice of \( A \) that replicates the Navier-Stokes equations of motion.
Consider the exterior differential 1-form, $A$, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $v = v_k(x, y, z, t)$, and a scalar potential function, $\phi$:

$$A = v \circ dr - \phi dt = v_k(x, y, z, t)dx^k - \phi dt.$$
Consider the exterior differential 1-form, $A$, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $v = v_k(x,y,z,t)$, and a scalar potential function, $\phi$:

$$A = v \circ dr - \phi dt = v_k(x,y,z,t)dx^k - \phi dt.$$  

Compute the exterior differential $dA$ and define the following functions as:

$$\omega = \text{curl } v, \quad a = +\left\{\frac{\partial v}{\partial t} + \text{grad}(\phi)\right\},$$

$$F = dA = \omega_x dx^y dy + \omega_y dy^z dz + \omega_z dz^x dx - a_x dx^y dt - a_y dy^z dt - a_z dz^x dt,$$

$$dF = 0 \supset \text{curl } (-a) + \partial \omega / \partial t = 0, \quad \text{div } \omega = 0. \text{ The Faraday induction PDE's.}$$
Consider the exterior differential 1-form, $A$, of Action per unit source (in fluids, the unit source is mole number, or sometimes mass), constructed from a covariant 3D velocity field, $\mathbf{v} = v_k(x, y, z, t)$, and a scalar potential function, $\phi$:

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In essence, the topological formulation of electrodynamic intensity fields and hydrodynamic intensity fields are identical except for notation. Using the notational equivalences,

$$A \iff \mathbf{v}, \quad \phi \iff \mathbf{v} \cdot \mathbf{v}/2,$$

$$E \iff -a, \quad \mathbf{B} \iff \omega.$$
permits the EM formats to be rewritten in hydrodynamic format. The **Work** 1-form for a fluid becomes,

\[-\rho \{-a + v \times \omega\} \, dr - \rho \{v \circ a\} \, dt = W.\]
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\[-\rho\{-a + v \times \omega\} \circ dr - \rho\{v \circ a\}dt = W.\]

Specialization of the topological properties of the Work 1-form lead to familiar formulations of hydrodynamics. For example, suppose that the PTD of $W$ is 1; then $W = -dP$. With this topological constraint, the system of PDE's are recognized to be those that describe the classic Eulerian fluid:

\[-\rho\{-a + v \times \omega\} \circ dr - \rho\{v \circ a\}dt = -dP,
\{\partial v/\partial t + \text{grad}(v \cdot v/2) - v \times \omega\} = -\text{grad}(P)/\rho,
\text{Power theorem} \quad \partial P/\partial t = \rho v \circ a.\]
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\{\partial \mathbf{v}/\partial t + \text{grad}(\mathbf{v} \cdot \mathbf{v}/2) - \mathbf{v} \times \omega\} = -\text{grad}(P)/\rho,\]

Power theorem $\partial P/\partial t = \rho \mathbf{v} \circ \mathbf{a}$.

The "Bernoulli function (~Pressure)", $P$, if it exists, must be a first integral (a process invariant),

$L(\rho \mathbf{V}_4)P = (i(\rho \mathbf{V}_4)dP = 0.$
Based on the Pfaff Topological Dimension of $W$

Thermodynamic **Reversible Processes** imply that the Heat 1-form, $Q$ is integrable. \[ Q^dQ = 0 \]

**Extremal:**

Hamiltonian

\[ \text{PTD}(W) = 0, \ W = 0, \]

**Bernoulli-Casimir:**

Hamiltonian

\[ \text{PTD}(W) = 1, \ W \text{ exact} \]

**Helmholtz:**

Conservation of Vorticity

\[ \text{PTD}(W) = 1, \ W \text{ closed} \]

Each of these flows are thermodynamically reversible, as

\[ dW = 0 = dQ, \quad \text{implies} \quad Q^dQ = 0. \]
In order to go beyond Hamiltonian or Bernoulli flows, it is necessary that the Pfaff Topological Dimension of the Work 1-form must be greater than 1. Recall that for any process, the Work done is transverse to the process trajectory,

\[(\iota(\rho \mathbf{V}_4) W = (\iota(\rho \mathbf{V}_4) (\iota(\rho \mathbf{V}_4) dA = 0).\]
In order to go beyond Hamiltonian or Bernoulli flows, it is necessary that the Pfaff Topological Dimension of the Work 1-form must be greater than 1. Recall that for any process, the Work done is transverse to the process trajectory,

\[(i(\rho V_4)W = (i(\rho V_4)(i(\rho V_4)dA = 0).\]

Hence, if the PTD of the Work 1-form, \(W\), is to be greater than 1, it must have the format,

\[W = i(\rho V_4)dA = -dP + \omega_j(dx^j - v^jdt) = -dP + \omega_j\Delta x^j,\]

where \(\Delta x^j\) represents the topological fluctuation about kinematic perfection. It is also important to remember that such non-zero contributions to the Work 1-form are due to the complex, isotropic Cartan Spinors, which are the eigen direction fields of the 2-form, \(F\).
The coefficients, $\omega_j$, of the topological fluctuations, $\Delta x^i$, act in the manner of Lagrange multipliers, and mimic the concept of system forces. If $\omega_j$ is defined (arbitrarily) as $\nu \text{curl curl} v$, then the spatial components of the thermodynamic Work 1-form, $W$, are constrained to yield the partial differential equations for a constant density Navier-Stokes fluid:

$$\frac{\partial V}{\partial t} + \text{grad} \frac{V^2}{2} - V \times \text{curl} V = - \text{grad} P/\rho - \nu \nabla^2 V$$
The coefficients, \( \varpi_j \), of the topological fluctuations, \( \Delta x^j \), act in the manner of Lagrange multipliers, and mimic the concept of system forces. If \( \varpi_j \) is defined (arbitrarily) as \( \nu \) curl curl \( v \), then the spatial components of the thermodynamic Work 1-form, \( W \), are constrained to yield the partial differential equations for a constant density Navier-Stokes fluid:

\[
\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v}^2/2 - \mathbf{v} \times \nabla \times \mathbf{v} = -\nabla P/\rho - \nu \nabla^2 \mathbf{v}
\]

This is one of many formal choices, but the choice demonstrates that the Navier-Stokes equations reside within the domain of non-equilibrium thermodynamics. QED
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Density variations can be included by adding a term $\lambda \text{ div}(v)$ to the potential $\{v \circ v/2\}$ to yield:

$$\partial V/\partial t + \text{grad } V^2/2 - V \times \text{curl } V = - \text{gradP}/\rho + \lambda \text{ grad div } V - \nu \nabla^2 V$$

Classically, $\nu = \text{shear viscosity}$, and $\lambda = (\mu_B - \nu)$ where $\mu_B = \text{Bulk viscosity}$
The next step is to compute the abstract Topological Torsion 3-form, $T_4 = A^\wedge dA$

using the functions that replicate the Navier-Stokes equations
It can be shown that the abstract

Topological Torsion 3-form, \( T_4 = A^dA \),

generates a thermodynamically irreversible process.

By inserting the functions that replicate the Navier-Stokes equations into the formula for \( T_4 \), it is possible to derive an example of a solution to the Navier-Stokes equations that is non-integrable, chaotic, and thermodynamically irreversible

or

TURBULENT
The abstract formulation of the Topological Torsion leads to the 4 component functions:

\[
T_4 = [-a \times v + \{v \circ v/2\} \text{ curl } v, (v \circ \text{curl } v)], \\
= [-a \times v + \{v \circ v/2\} \omega, (v \circ \omega)] = [T, h].
\]

Note that the topological torsion axial vector current persists even for Euler flows with zero vorticity, \(\omega=0\). Moreover if the flow is harmonic, the topological torsion axial vector still exists with a term proportional to the bulk viscosity, \(\mu_B\).
From the expression for the homotopic first law that replicates the Navier-Stokes equations, it is possible to solve for the acceleration, $a$.

$$a = [\text{grad}(v \circ v/2) + \partial v/\partial t]$$
$$= v \times \text{curl } v - \text{grad} P/\rho$$
$$+ \lambda \{\text{grad}(\text{div } v)\} + v \{\text{curl } \text{curl } v\},$$

Substitute the expression for $a$ into the equation for the components of the Topological Torsion 3-form,

$$T_4 = A^\wedge dA$$

$$T = [hv - (v \circ v/2)\text{curl } v - v \times (\text{grad} P/\rho)$$
$$+ \lambda \{v \times \text{grad}(\text{div } v)\} - v \{v \times (\text{curl } \text{curl } v)\}],$$
$$h = v \circ \text{curl } v,$$
For the Navier–Stokes fluid, the Topological Parity Dissipation Coefficient, $K = dA^dA$, if NOT zero, insures that flow-process $V$ is thermodynamically irreversible,

or equivalently, Turbulent, with a dissipation factor $K$:

$$K = -2(a \cdot \omega) = 2 \{\text{grad} P/\rho - \mu_B \text{grad div } V - \nu V \times \nabla^2 V\} \cdot \omega$$

For a Navier Stokes fluid, the universal Dissipation Coefficient, $K$, is the sum of Baroclinic forces, minus accelerations of expansion (Bulk viscosity) and accelerations of rotation (shear viscosity), times the flow vorticity, $\omega$.

For A Plasma $K = 2(E \cdot B)$

To minimize dissipation, the fluid acceleration and the fluid vorticity should be orthogonal.
Summing up, Category theory has shown how certain solutions of the Navier-Stokes Equations
\[
\frac{\partial V}{\partial t} + \text{grad} \, V^2/2 - V \times \text{curl} \, V = -\frac{dP}{\rho} + \mu_B \text{grad} \, \text{div} \, V - \nu \nabla^2 V
\]
generate Thermodynamically Irreversible Processes
and how these solutions may be used to give insight into Turbulence
In terms of a universal (topological) dissipation Coefficient, $K$
Part 2.
Topological, Non-Equilibrium Thermodynamics
And its Turbulent Artifacts
Examples of Turbulence

Turbulent recognition in terms of bubble generation and froth.
Turbulence and Topological Defects

The topological defects produced by turbulence are often associated with the evolution of deformation invariants, such as a evolutionary change of phase, or the number of parts.

Or the condensation from vapor to droplets, or the amalgamation of droplets into liquid, or the creation of wakes, vortices and solitons.
Turbulence and Topological Defects

In this presentation I would like to emphasize a common occurrence, which is the emergence caused by irreversible thermodynamic, turbulent processes, of Spiral Arms and a Tubular Vortex.
I was puzzled by the persistent long-lived Vortex Ring emergent from the turbulence of a nuclear explosion.
Vortex & Spiral Arm generation by turbulent flows Ex 1.

Note the persistent long-lived Vortex Ring emergent from the turbulence of a nuclear explosion.
Note the persistent long-lived ionized Vortex Ring emergent from the turbulence of a nuclear explosion

The Mushroom shape is an indicator of the Rayleigh-Taylor instability.

The spiral arms are rotated about z axis, and have Frenet Theory exact solutions.
Vortex & Spiral Arm generation by turbulent flows Ex 2.
Note the spiral arms in the boundary layer, between the water and air, leading into the water spout vortex core.
Vortex and Spiral Arms

Generated by Solutions to the equations of Continuous Topological Evolution

Note the spiral arms in the boundary layer, between the water and air, leading into the water spout vortex core.

\[ X := -\left( a + c e^{bt} \right) \sin(\omega t) \]
\[ Y := \left( a + c e^{bt} \right) \cos(\omega t) \]
Vortex and Spiral Arms

**Surprise**  Two Classes Of Solutions.

1. Those Solutions that decay by contraction from the exterior to the Limit Cycle.

2. A New Solution that decays by contraction from the exterior, Penetrates the Limit cycle, and then expands to the Limit Cycle from the interior.
Vortex generation by spiral turbulent flows

Tangential entry of warm air creates a Vortex Limit Cycle

Compare this patented device to the water spout of the previous slide
Consider a Dynamical System in a 2D Fluid

\[
U := -b \, x(t) - \omega \, y(t) \\
V := -b \, y(t) + \omega \, x(t)
\]

\[
x(t) := e^{(-b \, t)} \cos(\omega \, t) \\
y(t) := e^{(-b \, t)} \sin(\omega \, t)
\]

Generate Ubiquitous Logarithmic Spirals

But no limit cycles
The equations

\[ X := -(a + c e^{bt}) \sin(\omega t) \quad Y := (a + c e^{bt}) \cos(\omega t) \]

Generate Logarithmic Spirals with Limit Cycles
Vortex Cores and Spiral Wakes are artifacts of Turbulent Dissipation.

Spiral arms generated by turbulent wakes

Limit Cycle Core

Vortex Cores and Spiral Wakes are artifacts of Turbulent Dissipation.
The bulk of energy loss for an aircraft is due to the turbulent generation of tip vortices – Save energy-preserve the ecology.
Vortex & Spiral Arm generation by turbulent flows

Wake turbulence is a severe, local, ecological problem.
Vortex & Spiral Arm generation by turbulent flows

Note the persistent long-lived eye of the storm about the vortex core generated in the wake of a C5a
Note the spiral arms in the turbulent wake. The Spirals appear to be precursors of the vortex cores.
The Basic idea is that expansion and rotation are associated with flow fixed points.

It can be shown that the 3D dynamical system given be the equations

\[ V^x = \{ \pm \Omega y + (xg(r, z, a, b\ldots) \}, \]
\[ V^y = \{ \pm \Omega x + (yg(r, z, a, b\ldots) \}, \]
\[ V^z = f(r, z, \lambda, \alpha). \]

are solutions to the Navier-Stokes Equations in a Rotating Frame of reference,

“Some (new) closed form solutions to the Navier-Stokes equations”
http://www22.pair.com/csdn/pdf/nvsol2.pdf
Vortex & Spiral Arm generation by turbulent flows Ex 3.

A very simple subset of the formulas are given by the equations of a dynamical system:

\[
\begin{align*}
V_x &= Bx - \omega y \\
V_y &= By + \omega x \\
V_z &= Cz
\end{align*}
\]

B is the expansion parameter, \( \omega \) is the rotation parameter.

This dynamical system is a special case of a 3D Tertiary Hopf bifurcation (Langford).

\[
\begin{align*}
\text{Vorticity} &= [0, 0, 2 \omega] \\
\text{Divergence} &= 2B + C \\
\text{Helicity} &= 2Cz\omega
\end{align*}
\]

This subset of solutions generates the Ubiquitous Logarithmic Spirals.
Is the Universe mostly a Turbulent Gas??
With Stars and Galaxies as its defect condensates

Note the similarity between the spiral arm galaxy and the Hurricane. The independence from size is a topological property.
Hurricane Katrina, generated by turbulent vortex formation, killed or severely damaged 320 million large trees in Gulf Coast forests, which weakened the role the forests play in storing carbon from the atmosphere. The damage has led to these forests releasing large quantities of carbon dioxide into the atmosphere!
Vortex & Spiral Arm generation by turbulent flows Ex 5.

Dissipative turbulent generation of a persistent vortex. Ecological damage can be enormous.
Vortex & Spiral Arm generation  practical Apps

Ranque-Hilsch tube

Hot 100

Cold -30

ALSO Checkout the Windhexe machine at http://www.youtube.com/watch?v=xxuM7xWL5RQ
Summary of

Turbulence and its Visual Artifacts

Spiral Arms, Vortices, Wakes and Solitons

Are Residues of Turbulent Decay,

Which have EMERGED from a

Dissipative Open Thermodynamic Systems

Producing subsets of

Closed Non-Equilibrium Thermodynamic Systems
In a dynamical system

**Expansions and Rotations**

are associated with **Spiral Arms**

But a Topological Dimension of 3 is required for **Irreducible Thermodynamic Dissipation**

(hence 2D Turbulence is a Myth.)

The Existence of the Property of **TOPOLOGICAL TORSION**

Insures the topological dimension is 3 or more.
Frenet Equations, Spinors and Wakes
Using ideas generated by Continuous Topological Evolution

\[
\frac{d\mathbf{R}}{ds} = \begin{bmatrix}
\frac{dx}{ds} \\
\frac{dy}{ds}
\end{bmatrix} = \mathbf{t}(s) = \begin{bmatrix}
sin(Q(s)) \\
\cos(Q(s))
\end{bmatrix}.
\]

The unit tangent Vector, \( \mathbf{t} \)

\[
\frac{d\mathbf{t}(s)}{ds} = \kappa \mathbf{n}(s) = \{dQ/ds\} \begin{bmatrix}
\cos(Q(s)) \\
-\sin(Q(s))
\end{bmatrix},
\]
\[
\kappa = \{dQ/ds\}.
\]

The unit normal Vector \( \mathbf{n} \)

\[
\kappa = s^{-1}, \quad \kappa = s^0, \quad \kappa = s^1...
\]

: Log spiral, circle, Cornu-Fresnel spiral...

Spiral shapes
Frenet Equations, Spinors and Wakes
Using ideas generated by Continuous Topological Evolution

Flow past a sharp edge
Thanks for you ATTENTION

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Paperback Monographs available from
www.lulu.com/kiehn